FUNCTIONS AND GRAPHS

Precalculus

Chapter 1

- This Slideshow was developed to accompany the textbook
  - *Precalculus*
  - *By Richard Wright*
  - *https://www.andrews.edu/~rwright/Precalculus-RLW/Text/TOC.html*
- Some examples and diagrams are taken from the textbook.

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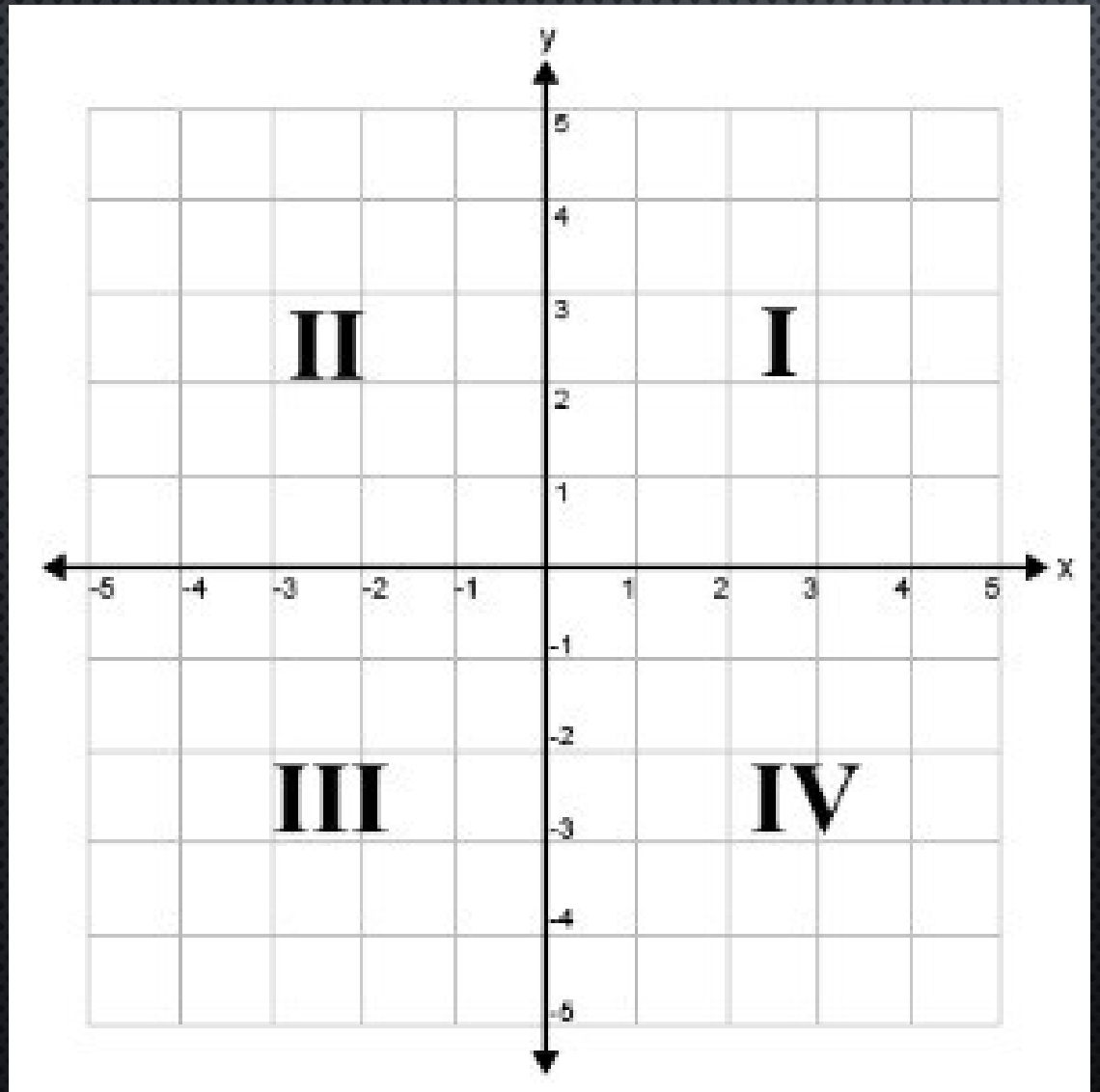
1-01 THE CARTESIAN PLANE

In this section, you will:

- Plot points in the cartesian coordinate system.
- Find the distance between two points.
- Find the midpoint between two points.

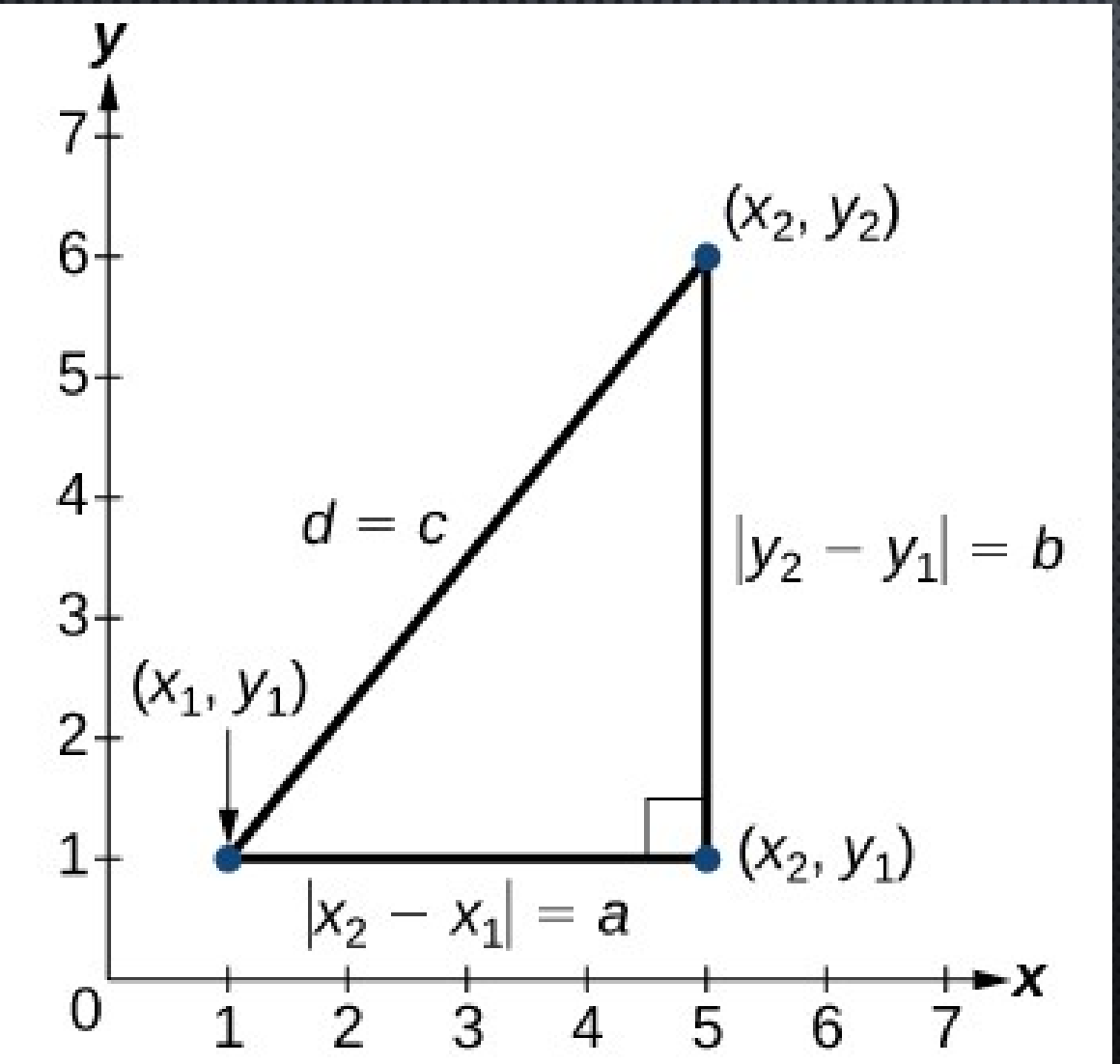
1-01 THE CARTESIAN PLANE



- Cartesian Plane
  - Four quadrants
- Point is (x, y)

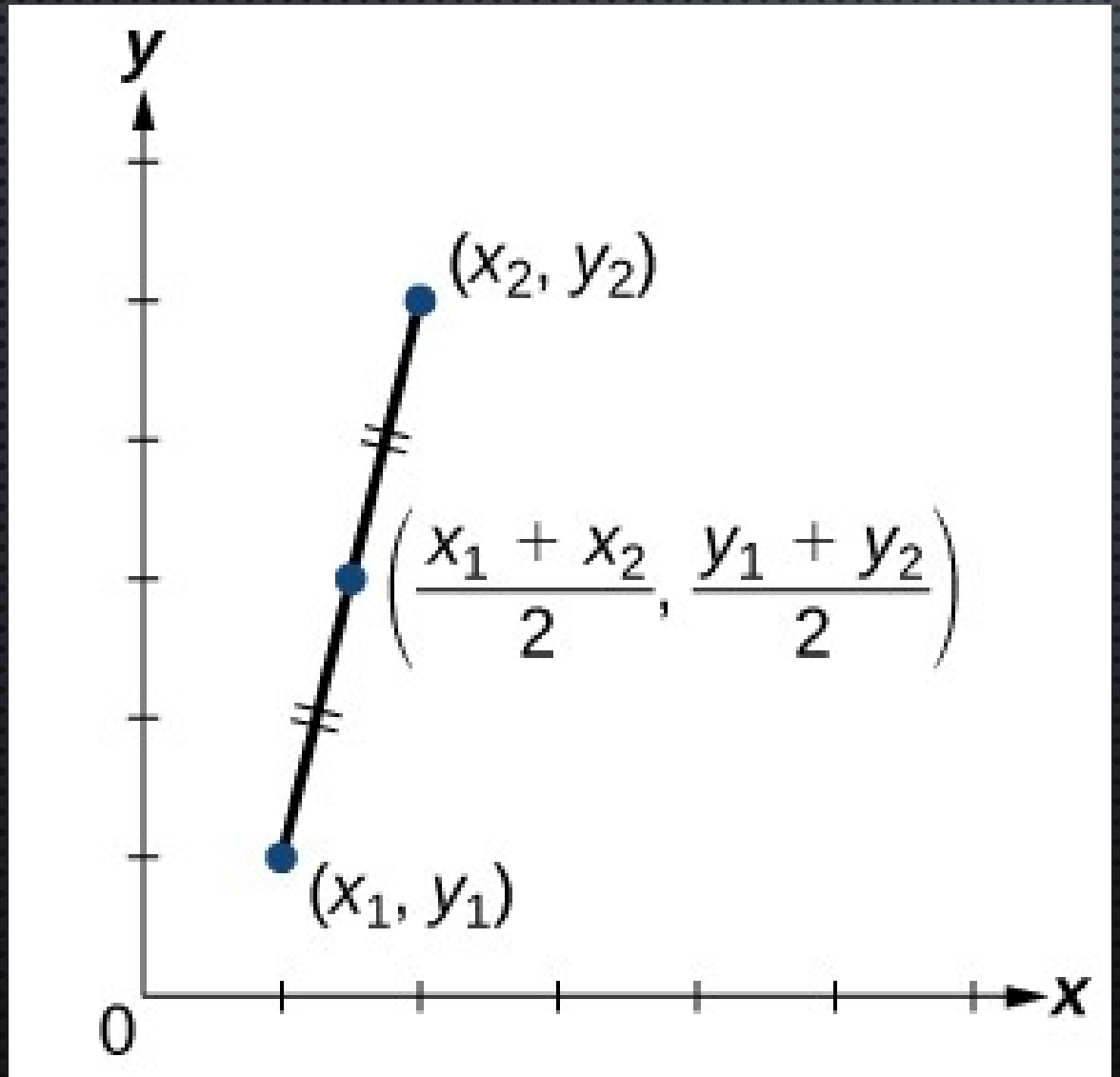
- Graph A(3, 2)
- Graph B(-1, 4)

1-01 THE CARTESIAN PLANE



- Distance formula
  - Pythagorean Theorem

1-01 THE CARTESIAN PLANE



- Midpoint formula
  - Average of the points (mean)
-

1-01 THE CARTESIAN PLANE

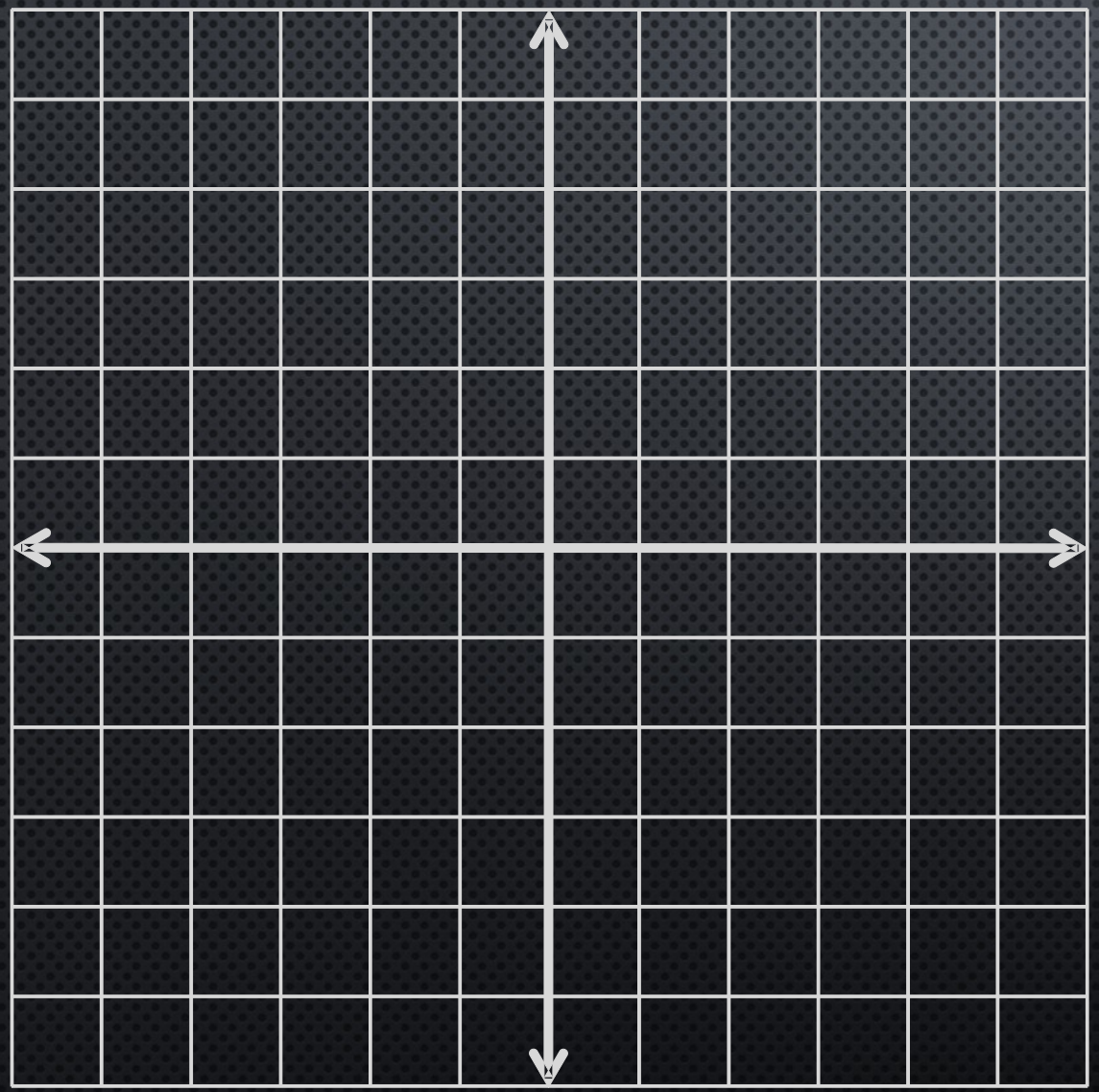
- Find the (a) distance and (b) midpoint between (-1, 3) and (2, -5)

1-02 GRAPHS

In this section, you will:

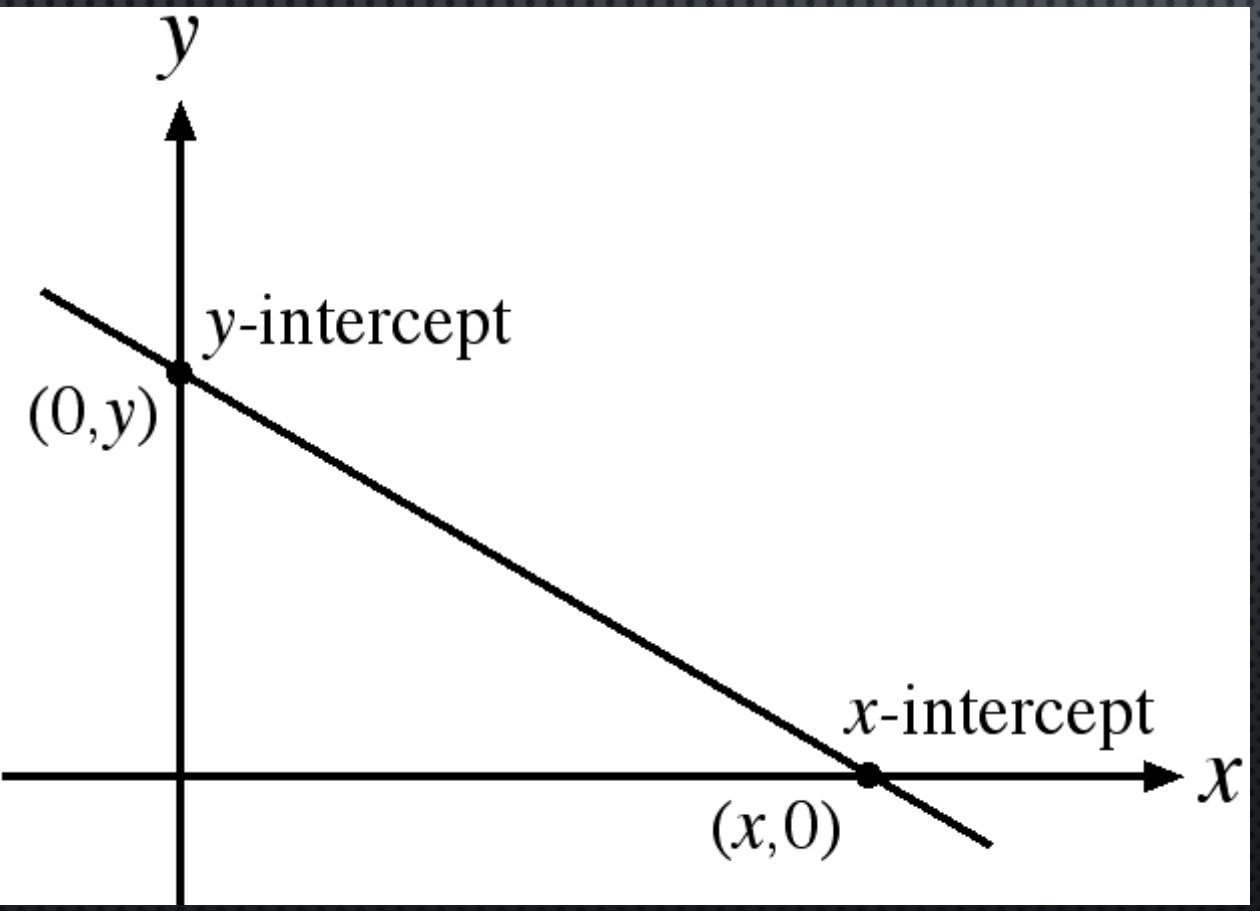
- Graph equations by plotting points.
- Graph equations with a graphing utility.
- Find the x- and y-intercepts.
- Graph circles.

1-02 GRAPHS



- Basic graphing method
  - Make a table
  - Choose x, Calculate y
- Graph

1-02 GRAPHS



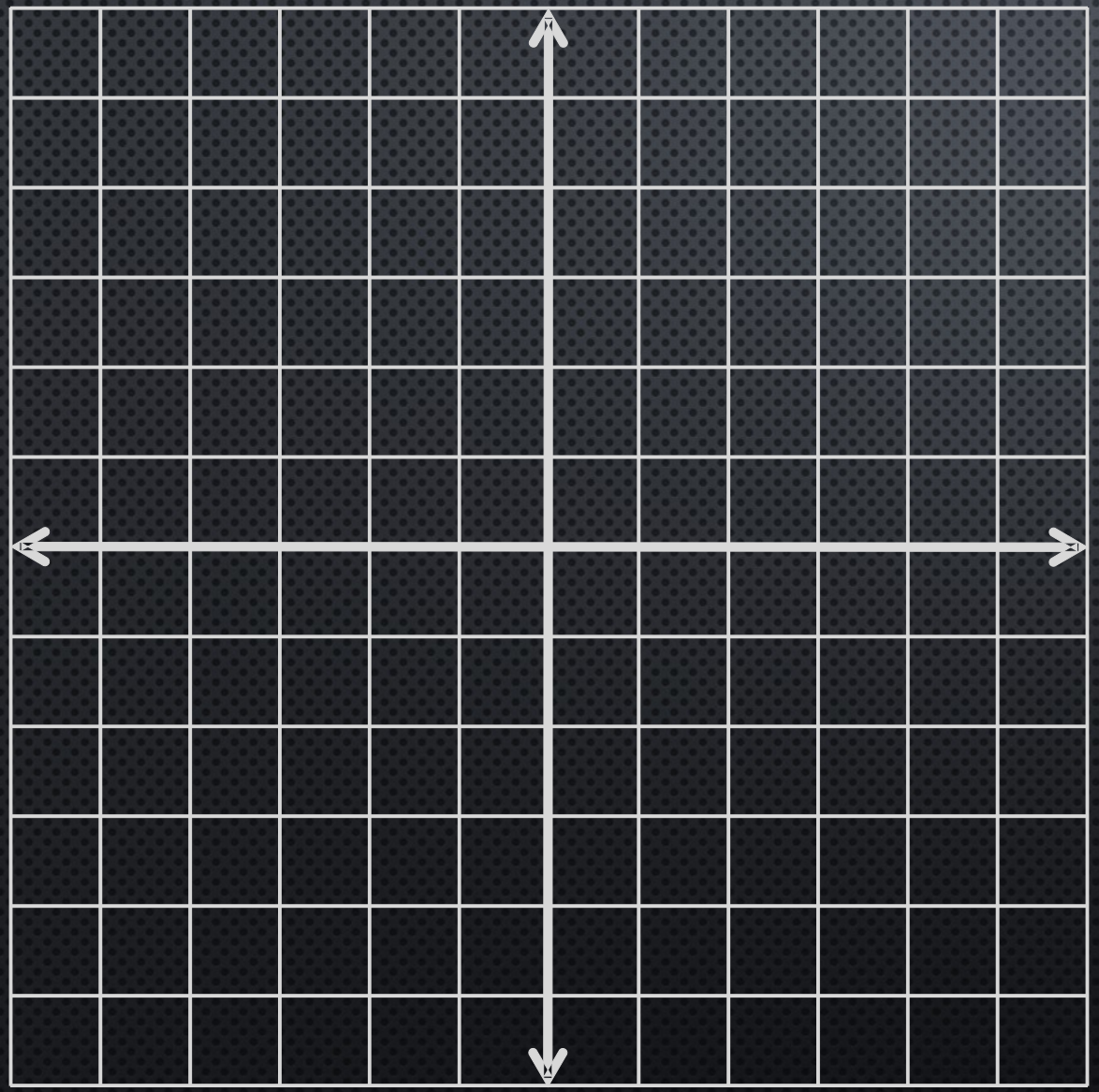
- Intercepts
- Point where a graph crosses the axes

- To find the intercepts
- *x*-intercept
  - Let and solve for *x*
- *y*-intercept
  - Let and solve for *y*

1-02 GRAPHS

- Find the intercepts of

1-02 GRAPHS



- Circles
-
  - where (h, k) is the center
  - and r is the radius
- Graph

1-03 LINEAR EQUATIONS IN TWO VARIABLES

In this section, you will:

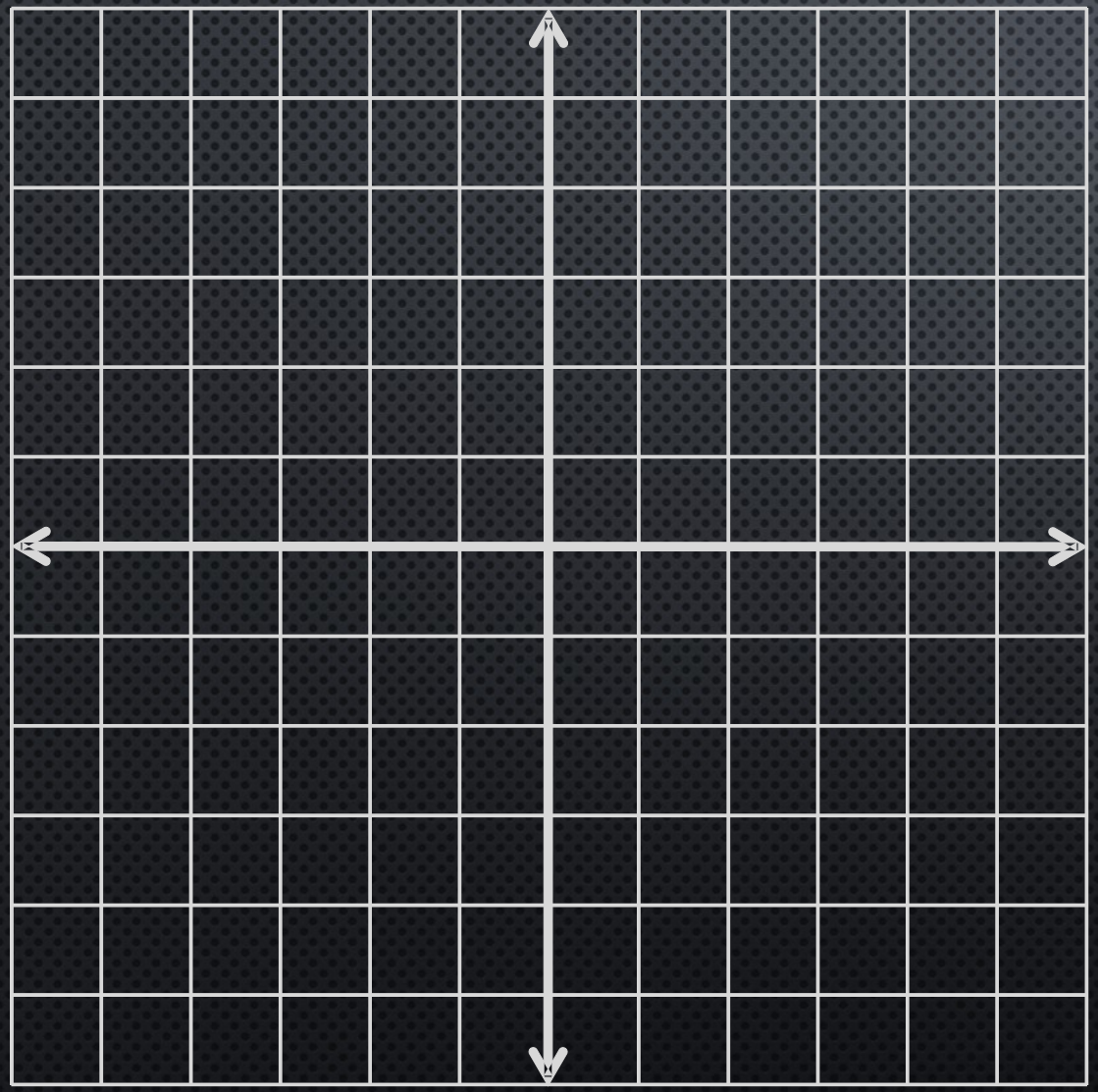
- Calculate and interpret slope.
- Write linear equations.
- Graph linear functions.

1-03 LINEAR EQUATIONS IN TWO VARIABLES

- Slope-intercept form
  - m = slope (rate of change)
  - $(0, b)$ = y-intercept
- □ horizontal line
- □ vertical line

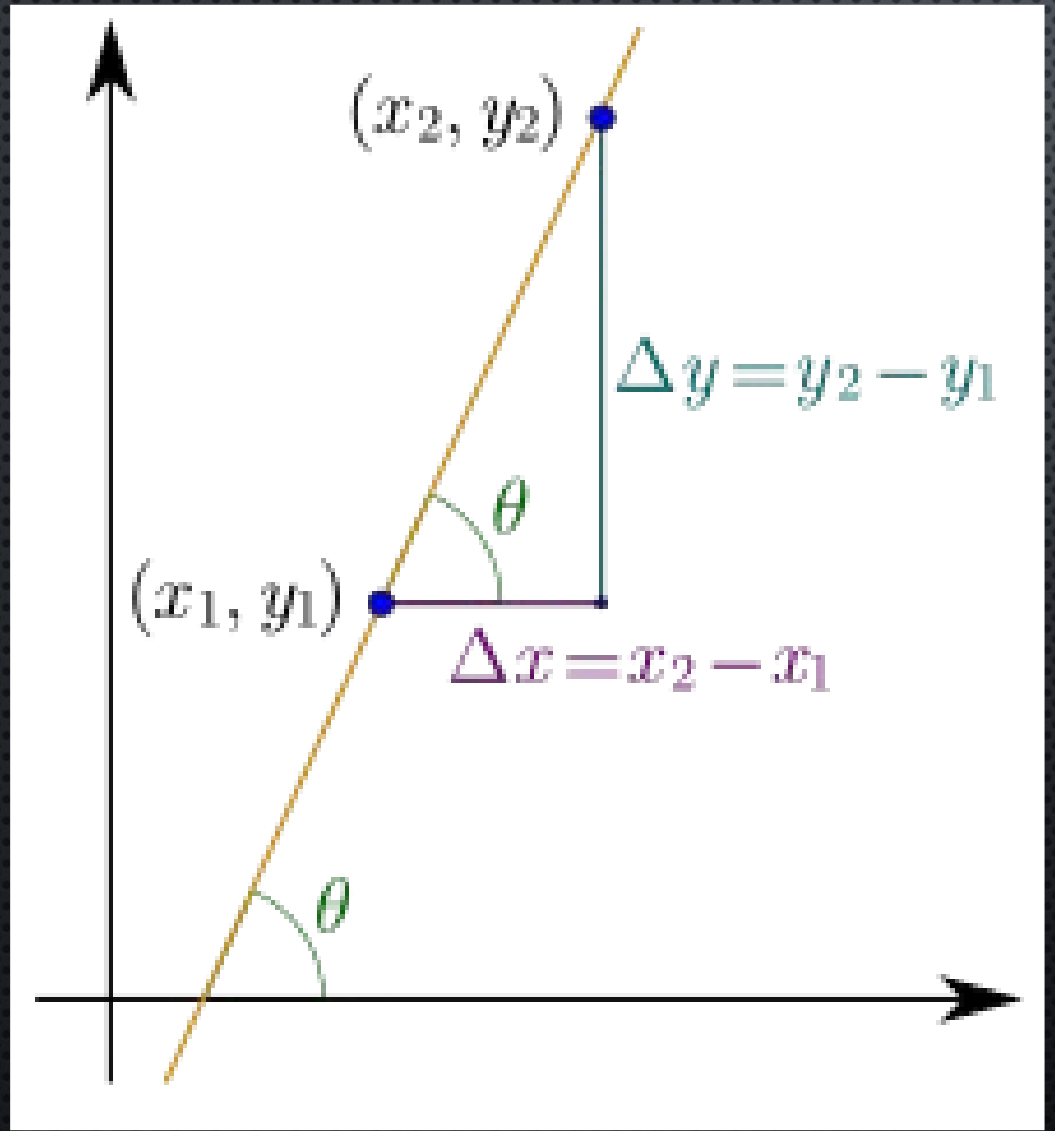
- To graph a line (shortcut)
  1. Plot y-intercept
  2. Follow the slope to get a couple more points
  3. Draw a line through the points

1-03 LINEAR EQUATIONS IN TWO VARIABLES



- Find the slope and *y*-int and graph

1-03 LINEAR EQUATIONS IN TWO VARIABLES



- Slope

- If slope is
  - $m > 0 \rightarrow$ rises
  - $m = 0 \rightarrow$ horizontal
  - $m < 0 \rightarrow$ falls
  - m undefined $\rightarrow$ vertical

1-03 LINEAR EQUATIONS IN TWO VARIABLES

- Find the slope of the line passing through (-3, -2) and (1, 6)

1-03 LINEAR EQUATIONS IN TWO VARIABLES

- Write Linear Equations

1. Find slope (m)
2. Find a point on the line
3. Use point-slope form

- Find slope-intercept form of the line passing through (0, -2) with $m = 3$.

1-03 LINEAR EQUATIONS IN TWO VARIABLES

- Parallel and Penpendicular
- Parallel → same slope
- Perpendicular → slopes are negative reciprocals
  -

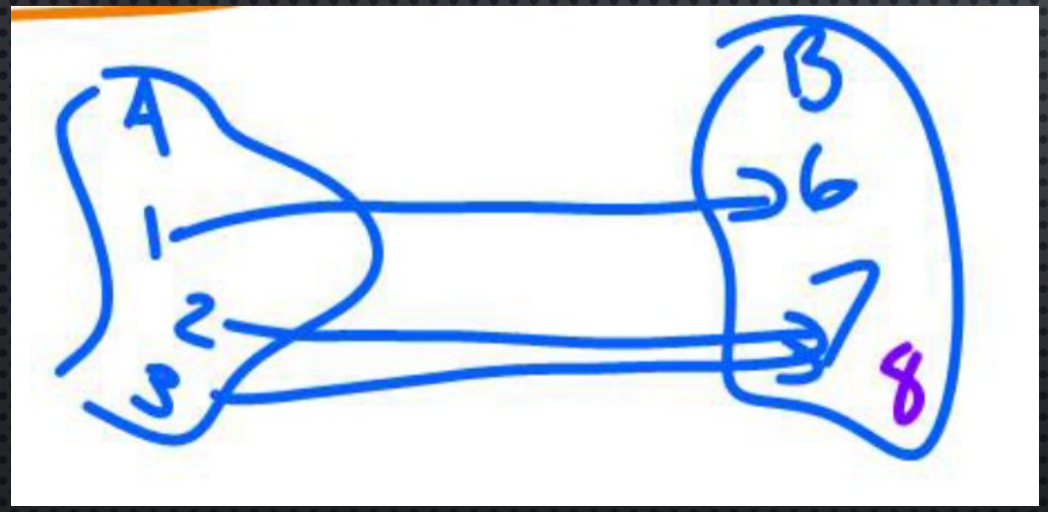
- Find the equation of the line passing through (2, 1) and perpendicular to .

1-04 FUNCTIONS AND FUNCTIONAL NOTATION

In this section, you will:

- Determine whether a relation represents a function.
- Find input and output values of a function.
- Find the domain of a function.
- Evaluate piecewise functions.

1-04 FUNCTIONS AND FUNCTIONAL NOTATION



- Relation
  - Rule that relates 2 quantities
- Function
  - Special relation
  - A function *f* from set A to set B is a relation that assigns each element *x* in set A to exactly one element in set B
  - Set A: input domain
  - Set B: output range

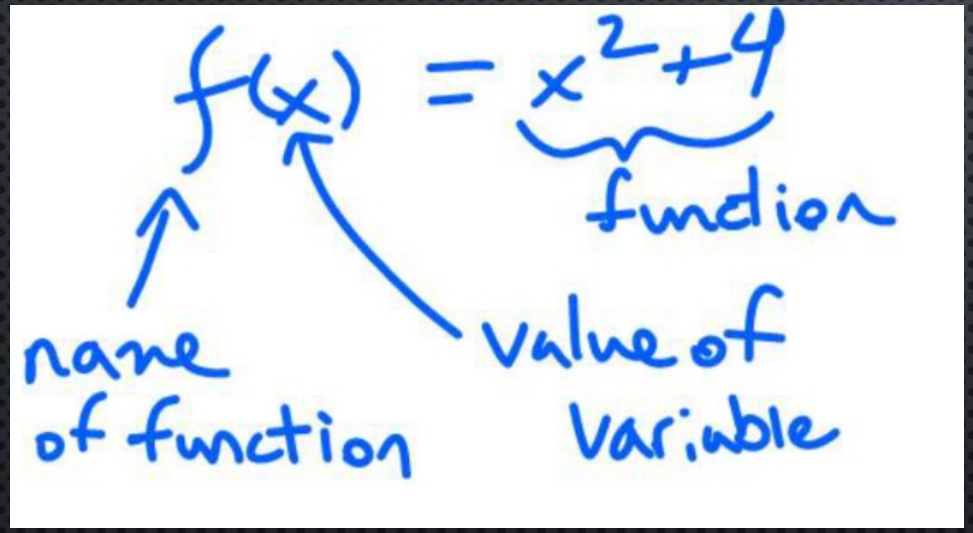
1-04 FUNCTIONS AND FUNCTIONAL NOTATION

- Is this a function?

x	-2	-1	0	1	2
y	-8	-1	0	1	8

1-04 FUNCTIONS AND FUNCTIONAL NOTATION

- Functional Notation



- Evaluate
-

1-04 FUNCTIONS AND FUNCTIONAL NOTATION

- Piecewise functions
  - Function made of more than one function with specific domains
- Evaluate

1-04 FUNCTIONS AND FUNCTIONAL NOTATION

- Domain of a function
  - Implied domain - all real numbers for which the expression is defined
- Interval notation
  - [ ] means =
  - ( ) means ≠
  - (2, 7] means $2 < x \leq 7$

- What is the domain?
-

1-04 FUNCTIONS AND FUNCTIONAL NOTATION

- Difference Quotient

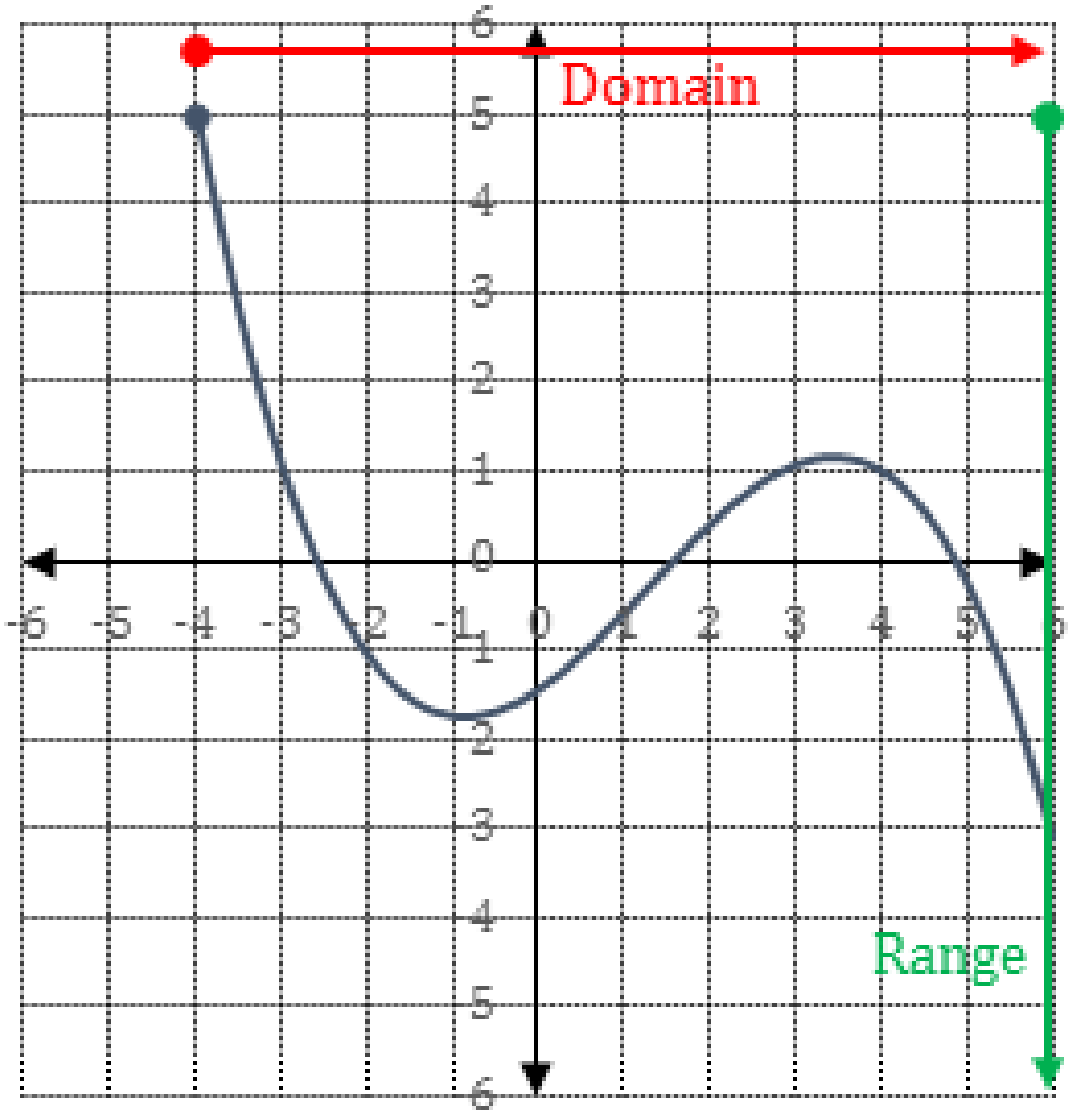
- Simplify the difference quotient for

1-05 GRAPHS OF FUNCTIONS

In this section, you will:

- Find domain and range from graphs.
- Determine whether graphs represent functions.
- Find zeros of functions.
- Find the average rate of change of a function.
- Analyze graphs to determine when the graph is increasing, decreasing, or constant.

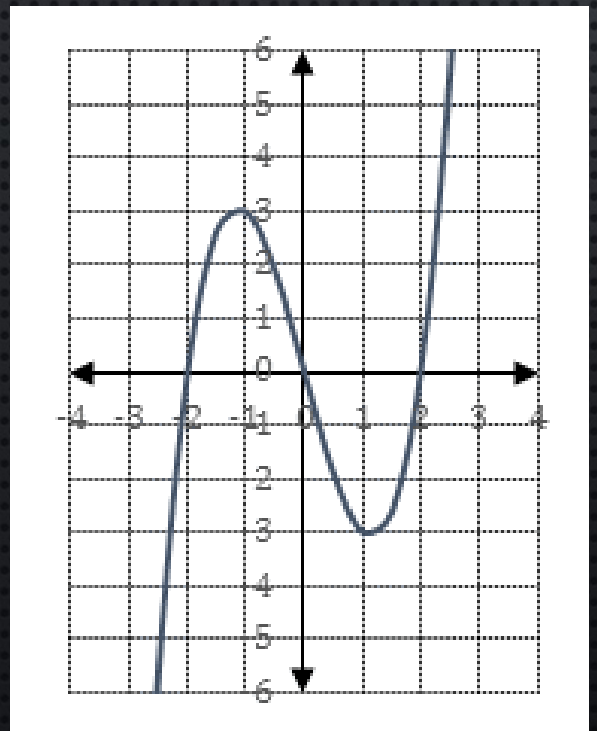
1-05 GRAPHS OF FUNCTIONS

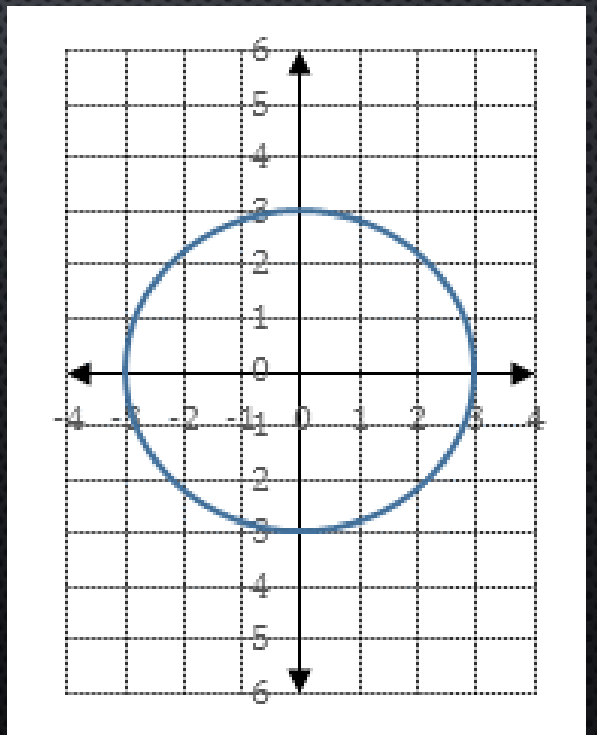


- Find the domain and range from a graph
- Domain: part of x-axis covered by graph
- Range: part of y-axis covered by graph

1-05 GRAPHS OF FUNCTIONS

- Vertical Line Test
- A graph represents a function if no vertical line can touch 2 points on the graph





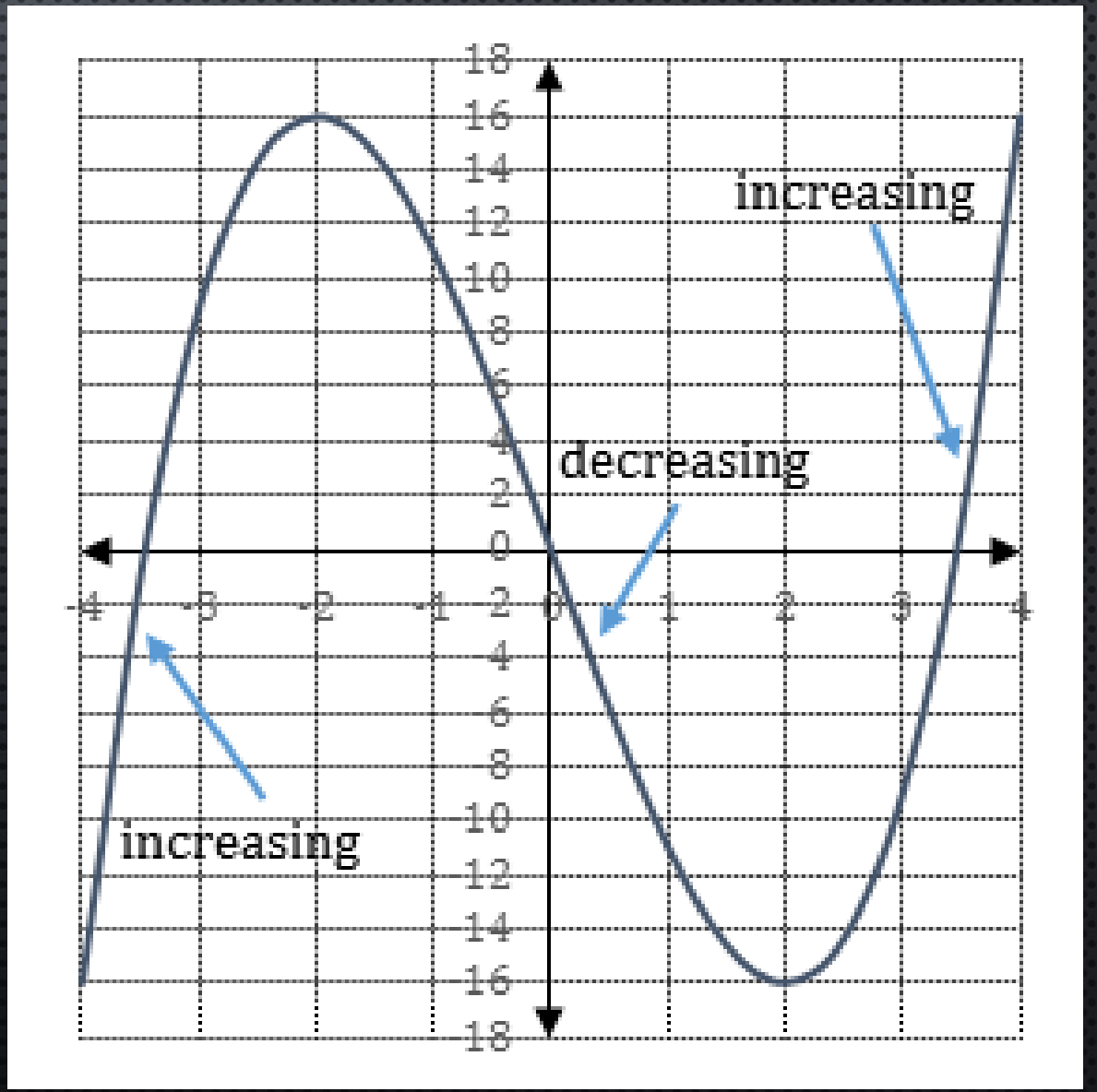
1-05 GRAPHS OF FUNCTIONS

- Zeros of a function
  - x-value such that
  - x-intercepts
- To find, make and solve for x

- Find the zeros of

1-05 GRAPHS OF FUNCTIONS

- Increasing (rises from left to right)
- Decreasing (falls from left to right)
- Constant (horizontal)
- Relative minimum (lowest point in area)
- Relative maximum (highest point in area)



1-05 GRAPHS OF FUNCTIONS

- Rate of Change
- Average rate of change = slope between 2 points

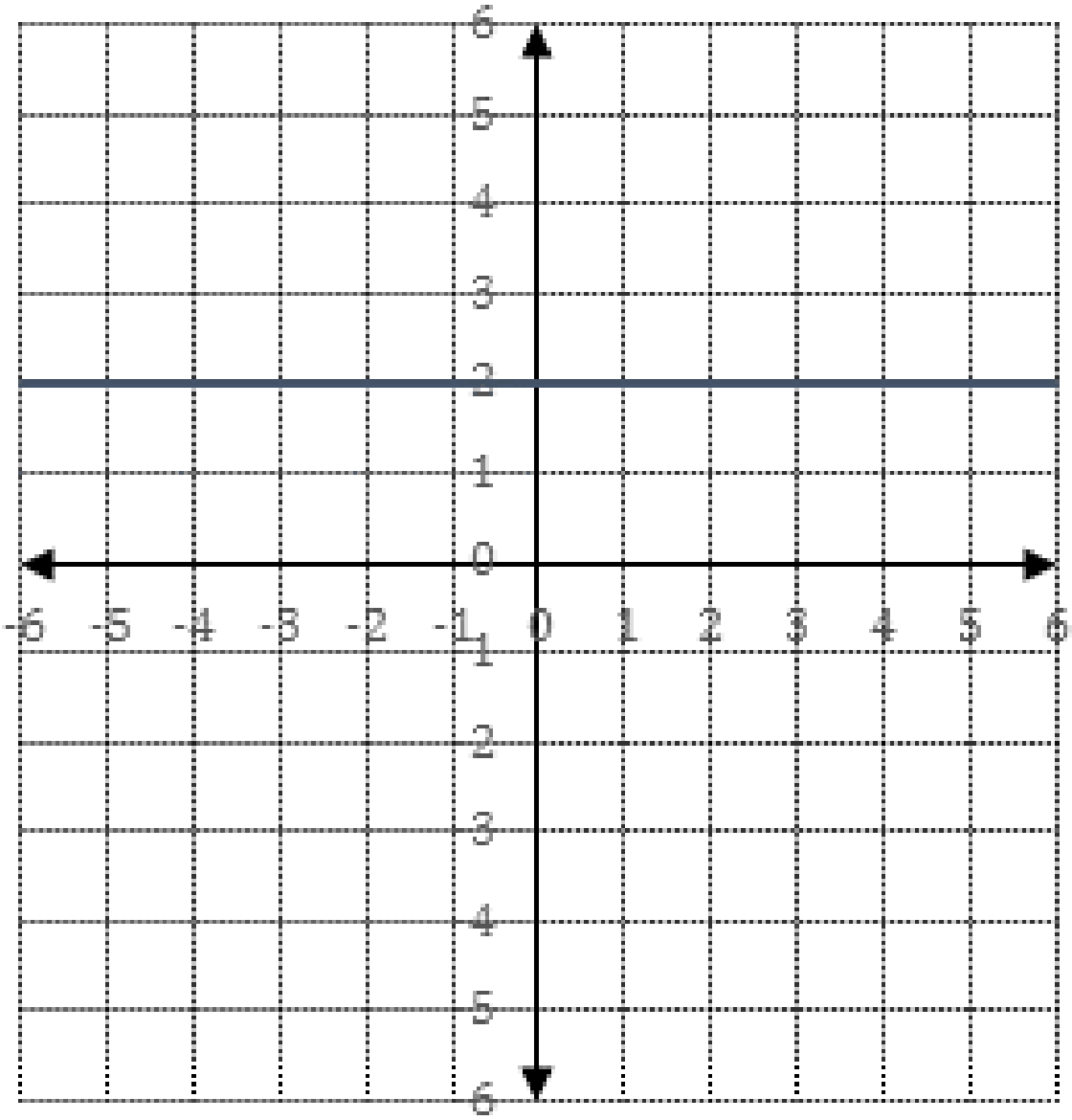
1-06 GRAPHS OF PARENT FUNCTIONS

In this section, you will:

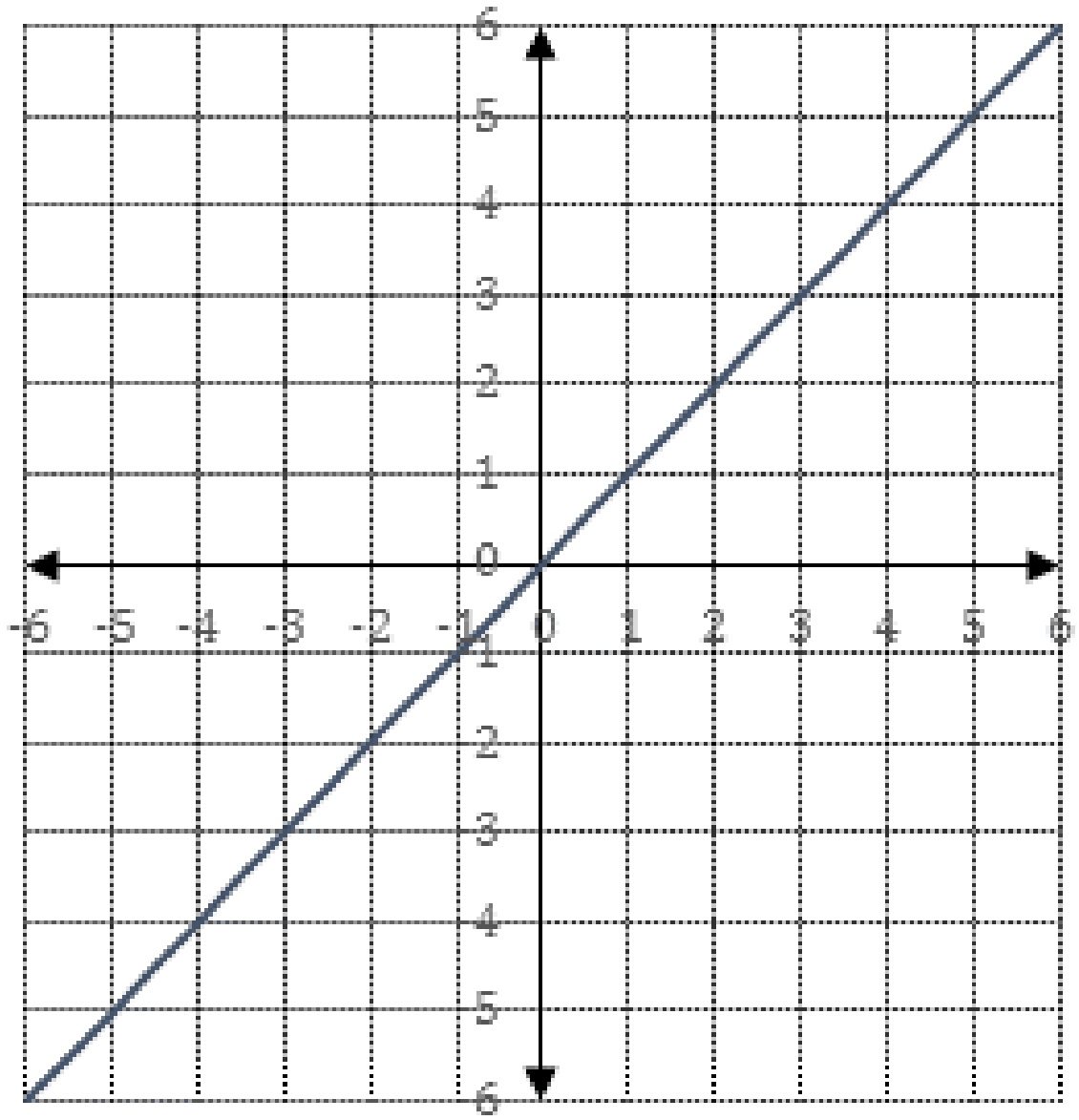
- Identify the graphs of parent functions.
- Graph piecewise functions.

1-06 GRAPHS OF PARENT FUNCTIONS

- **constant** function $f(x) = c$,
- Domain is all real numbers.
- Range is the set $\{c\}$ that contains this single element.
- Neither increasing or decreasing.
- Symmetric over the *y*-axis



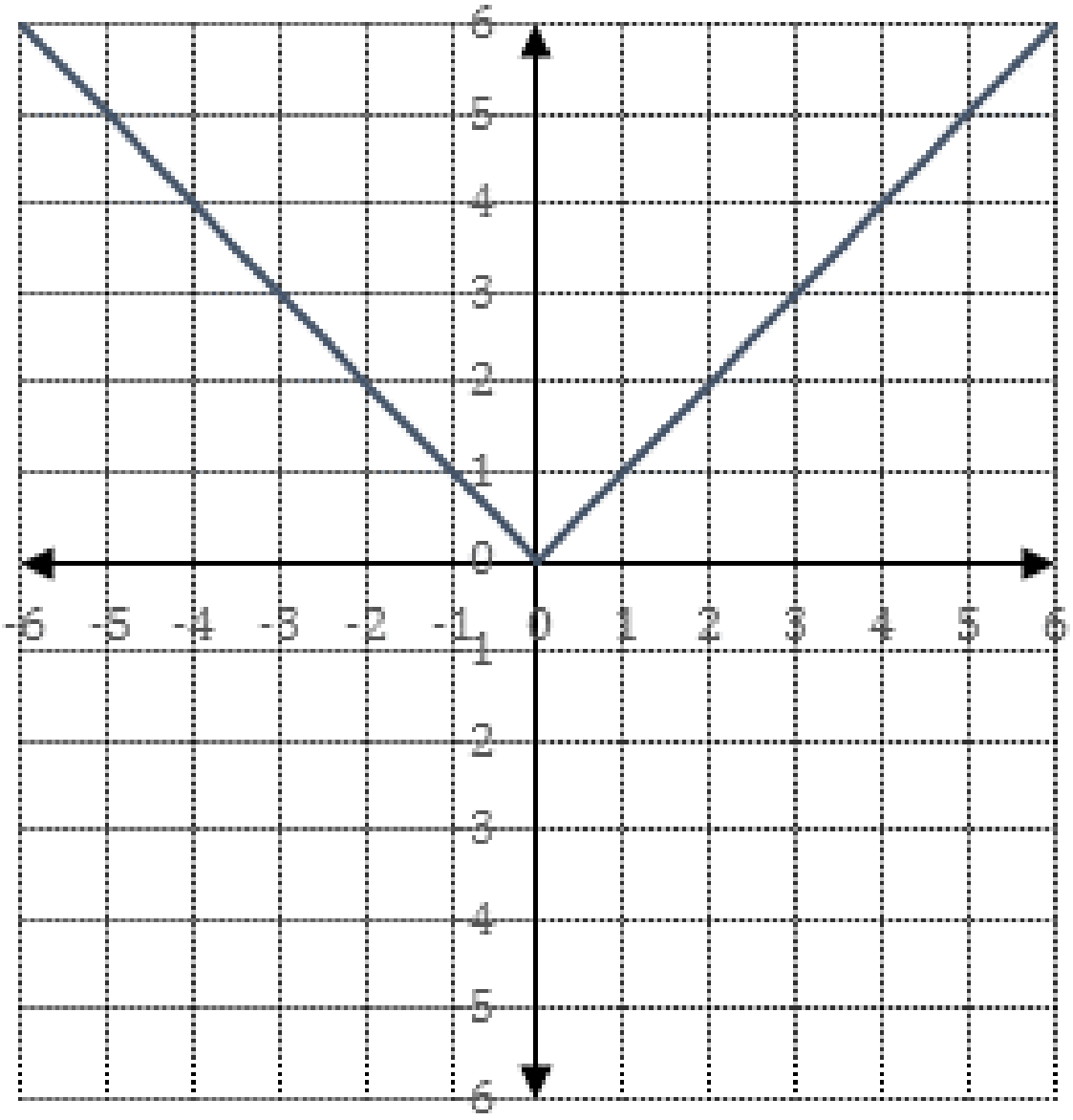
1-06 GRAPHS OF PARENT FUNCTIONS



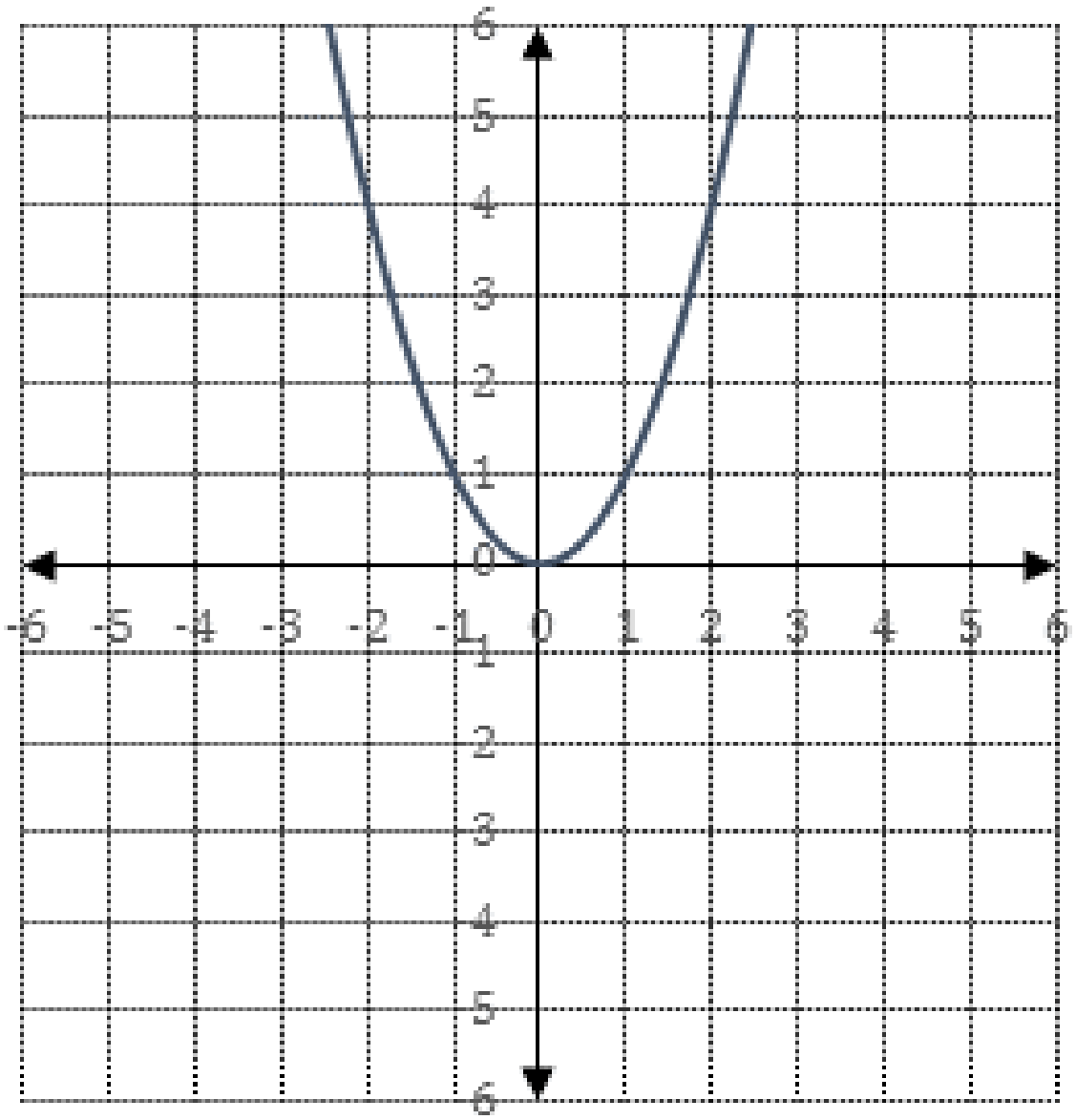
- **identity** function $f(x) = x$,
- Domain is all real numbers.
- Range is all real numbers.
- Increases from $(-\infty, \infty)$.
- Symmetric about the origin.

1-06 GRAPHS OF PARENT FUNCTIONS

- **absolute value** function
  ,
- Domain is all real numbers.
- Range is [0, ∞).
- Decreasing on (−∞, 0) and increasing on (0, ∞).
- Symmetric over the *y*-axis



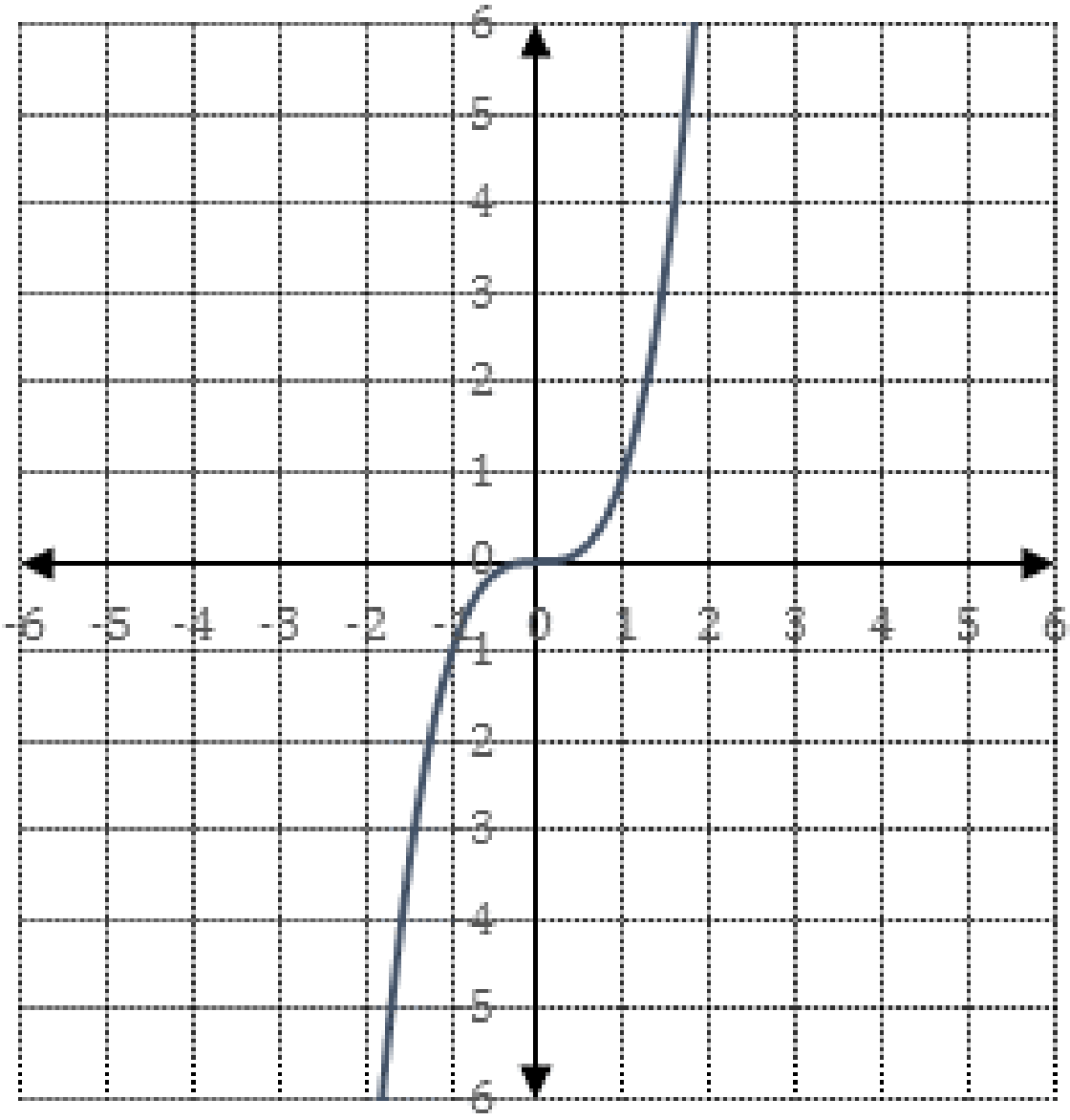
1-06 GRAPHS OF PARENT FUNCTIONS



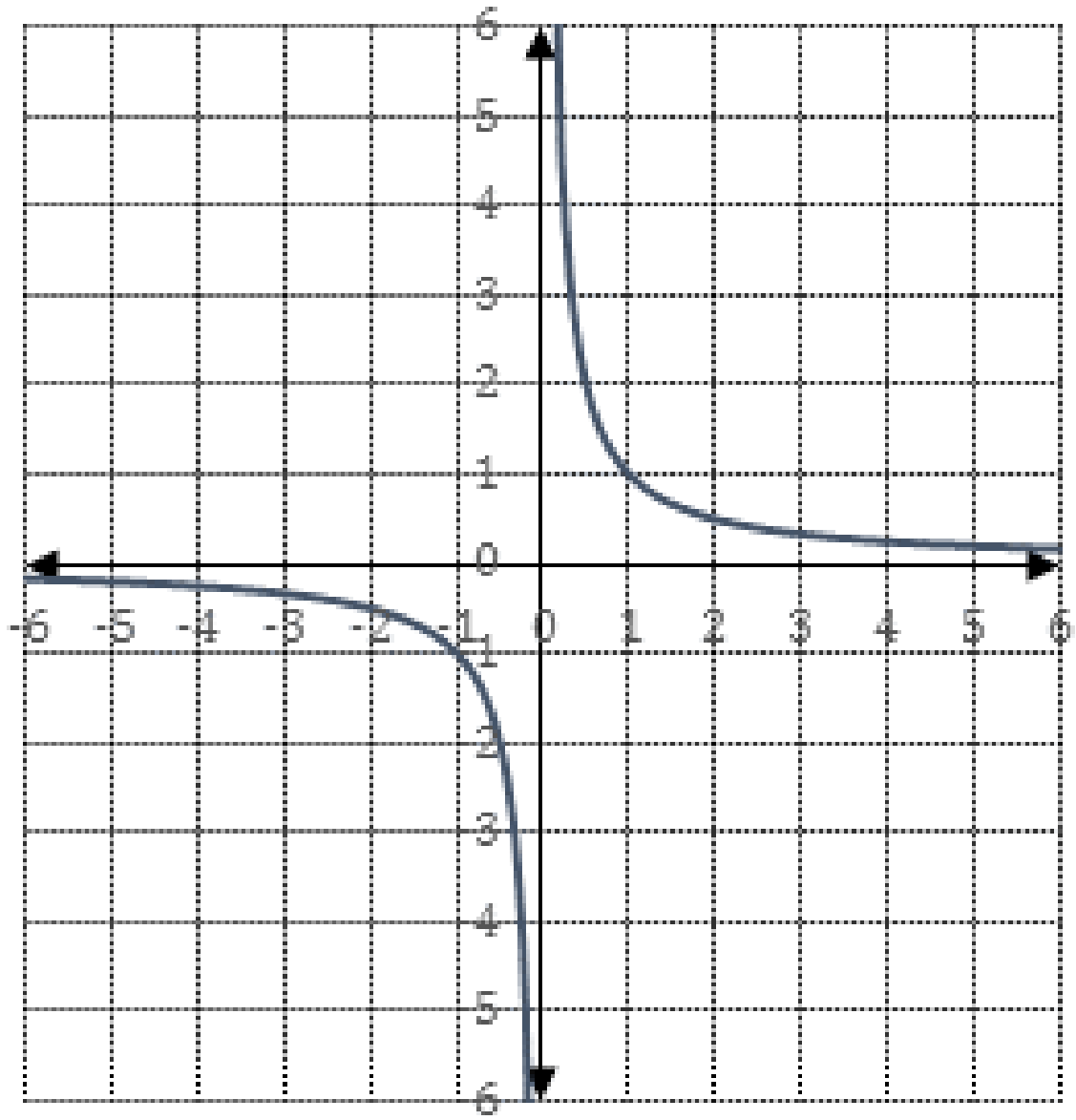
- **quadratic** function ,
- Domain is all real numbers.
- Range is only nonnegative real numbers, [0, ∞).
- Decreasing over (−∞, 0) and increasing on (0, ∞).
- Symmetric over the *y*-axis.

1-06 GRAPHS OF PARENT FUNCTIONS

- **cubic** function ,
- Domain is all real numbers.
- Range is all real numbers.
- Increasing on $(-\infty, \infty)$.
- Symmetric about the origin.



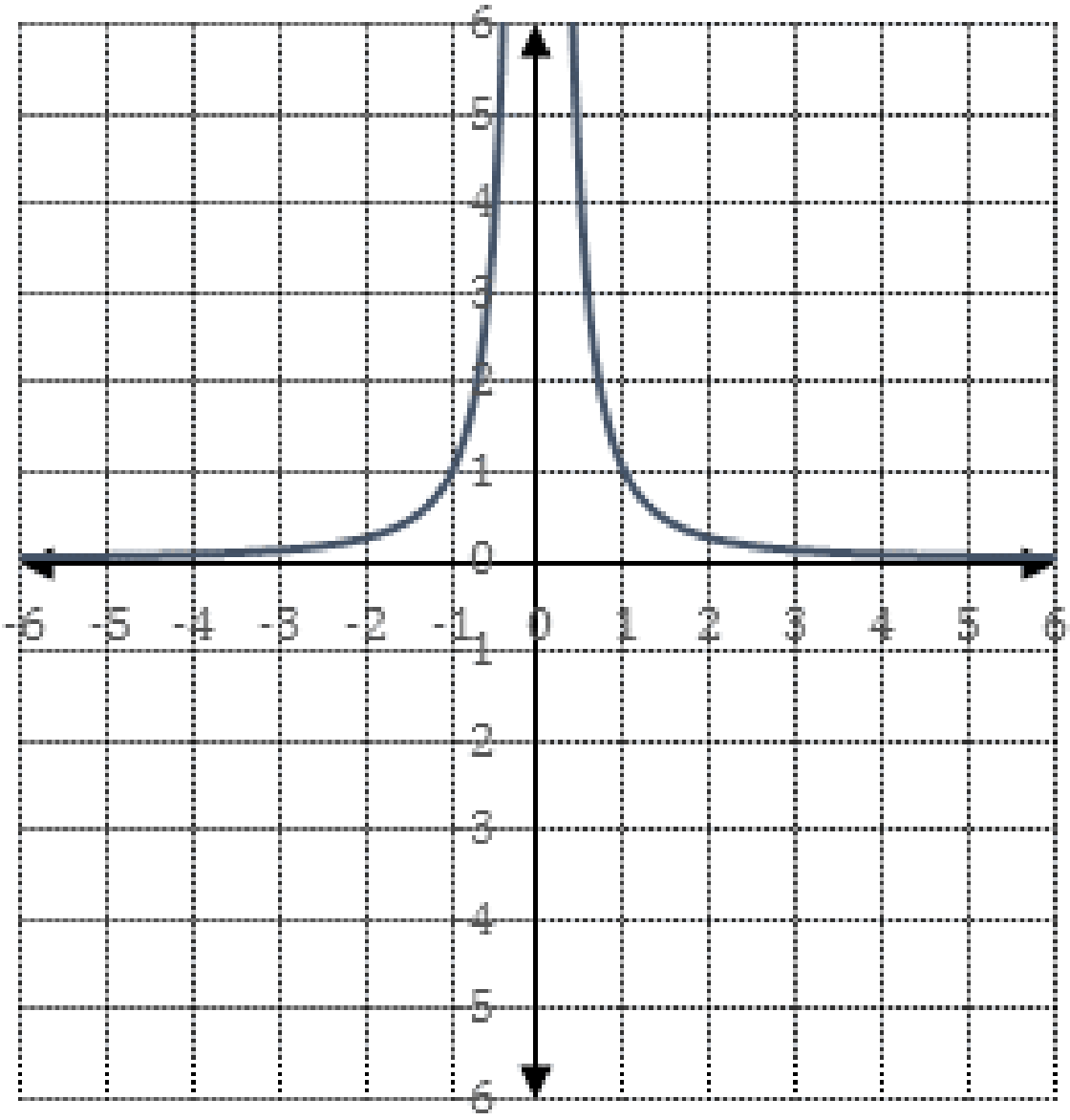
1-06 GRAPHS OF PARENT FUNCTIONS



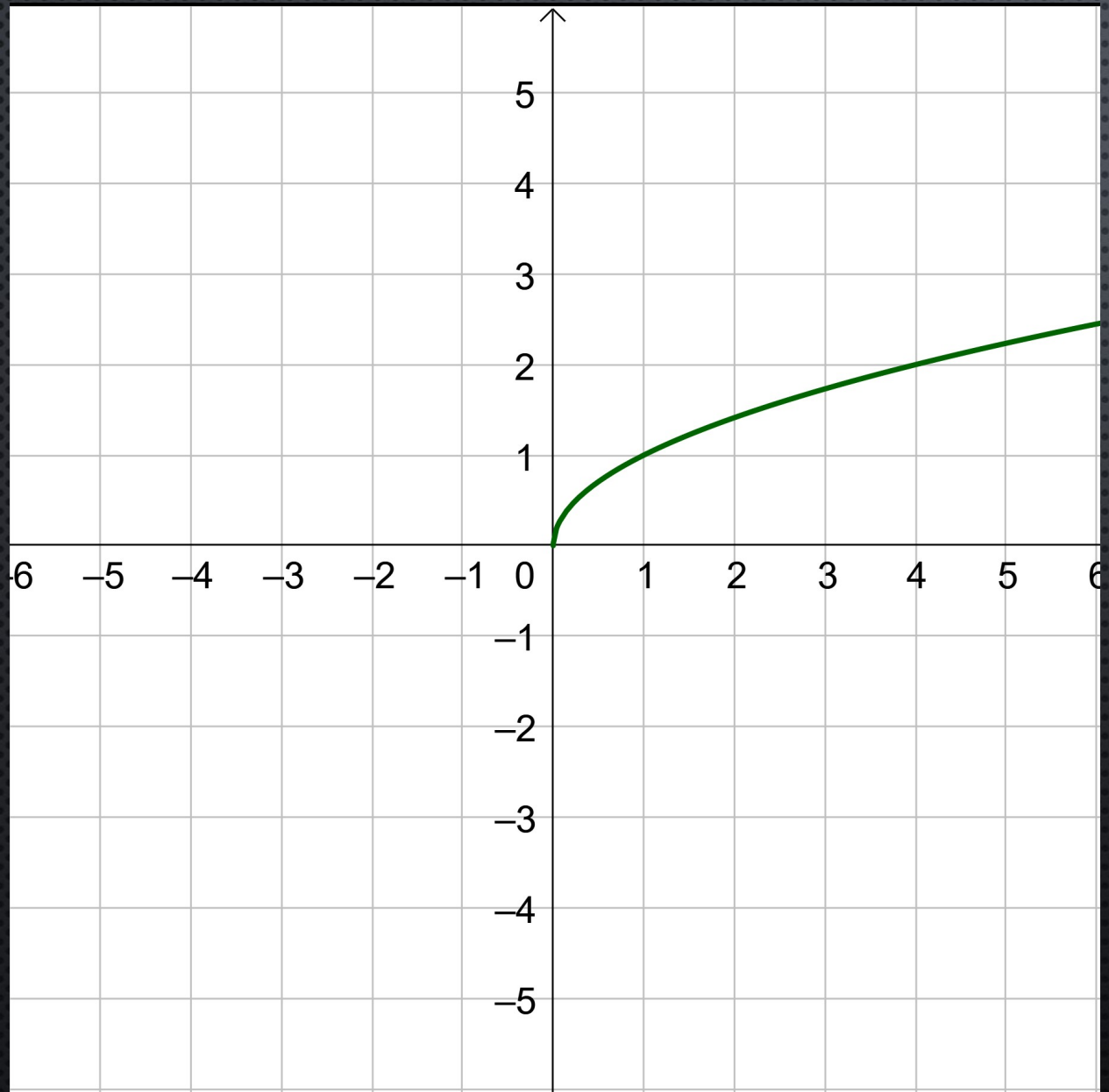
- **reciprocal** function ,
- Domain is all real numbers except 0, $\{x|x \neq 0\}$.
- Range is all real numbers except 0, $\{y|y \neq 0\}$.
- Decreasing on $(-\infty, 0)$ and $(0, \infty)$.
- Symmetric about the origin and over the lines $y = x$ and $y = -x$.

1-06 GRAPHS OF PARENT FUNCTIONS

- **reciprocal squared** function ,
- Domain is all real numbers except 0, $\{x|x \neq 0\}$.
- Range is only positive real numbers, $(0, \infty)$.
- Increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$.
- Symmetric over the y-axis.



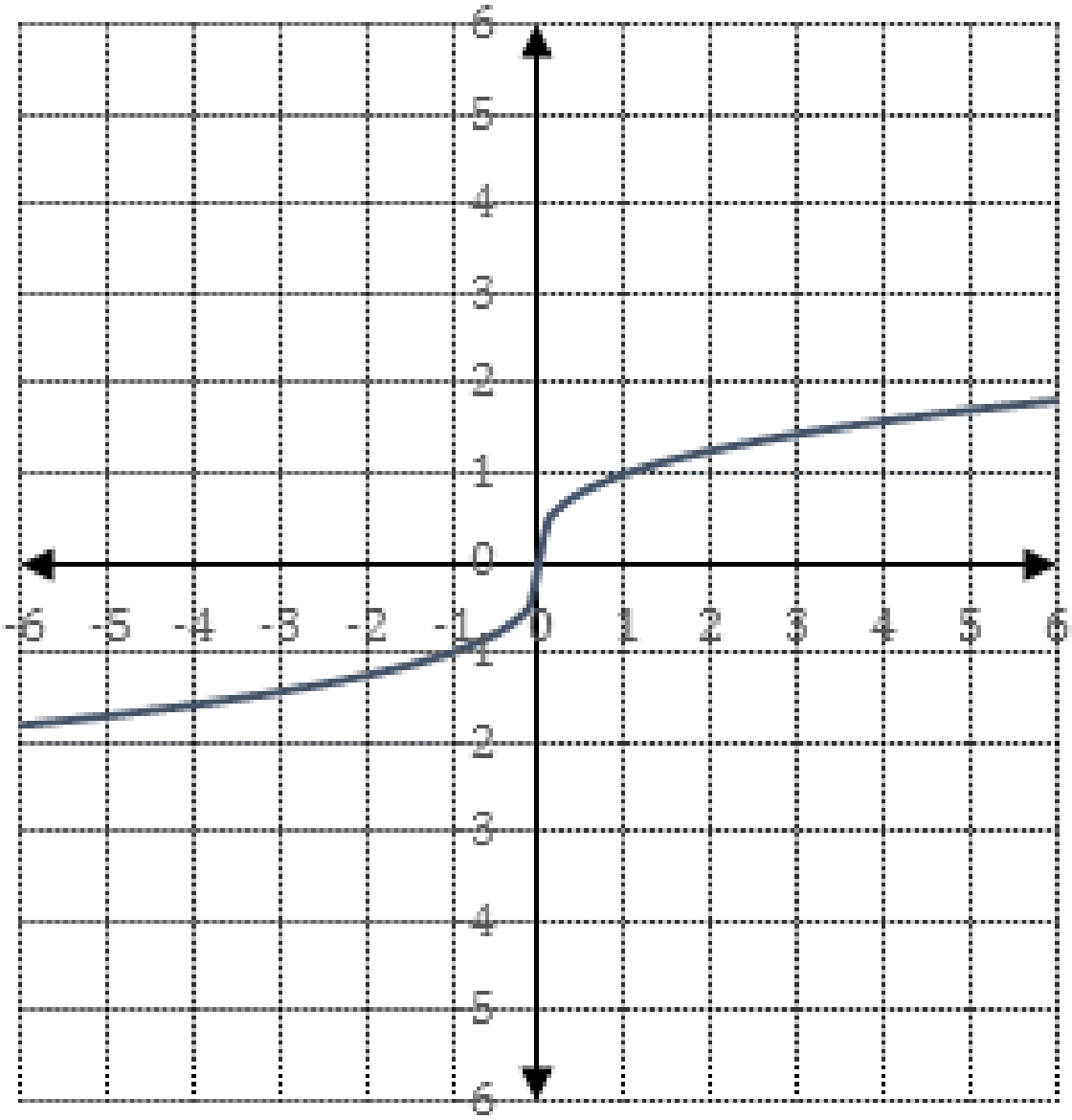
1-06 GRAPHS OF PARENT FUNCTIONS



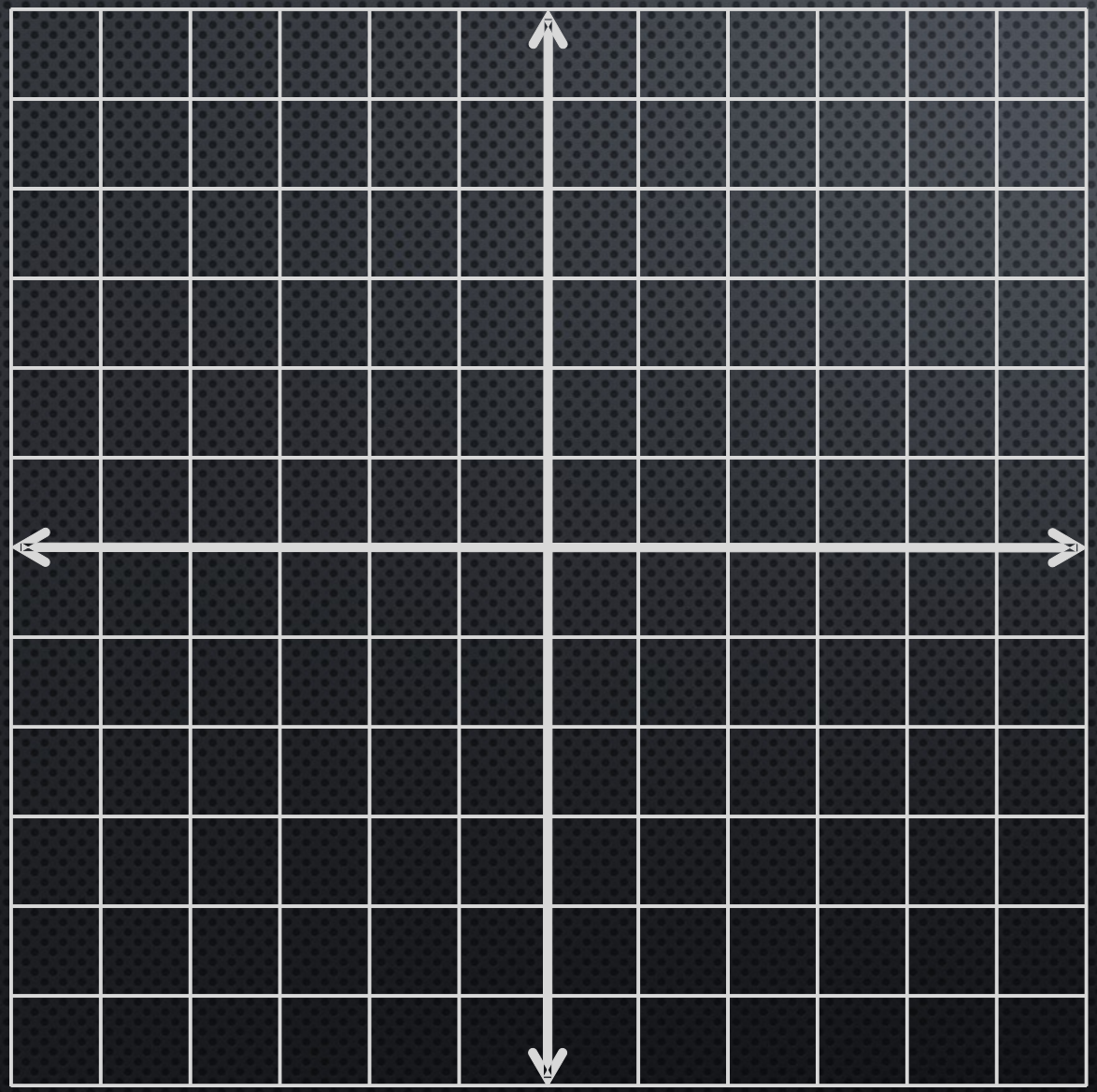
- **square root** function ,
- Domain is 0 or greater, [0, ∞).
- Range is 0 or greater, [0, ∞).
- Increasing on (0, ∞).
- No symmetry.

1-06 GRAPHS OF PARENT FUNCTIONS

- **cube root** function ,
- Domain is all real numbers.
- Range is all real numbers.
- Increasing over $(-\infty, \infty)$.
- Symmetric about the origin.



1-06 GRAPHS OF PARENT FUNCTIONS



- Piecewise Functions
- At the boundary,
  - If equal → solid dot
  - If not equal → open dot
- Graph

1-07 TRANSFORMATIONS OF FUNCTIONS

In this section, you will:

- Graph functions with translations.
- Graph functions with reflections.
- Graph functions with stretches and shrinks.
- Perform a sequences of transformations.

1-07 TRANSFORMATIONS OF FUNCTIONS

- Translations (shift)
  - Moves the graph
- Horizontal
  - c shifts right
- Vertical
  - d shifts up

- For , write a function with a vertical shift of 3 down and 2 right.

1-07 TRANSFORMATIONS OF FUNCTIONS

- Reflections
- x-axis
  - Vertical
- y-axis
  - Horizontal
- Dilations
  - Stretch/Shrink
- Horizontal
  - Stretch by
- Vertical
  - Stretch by a

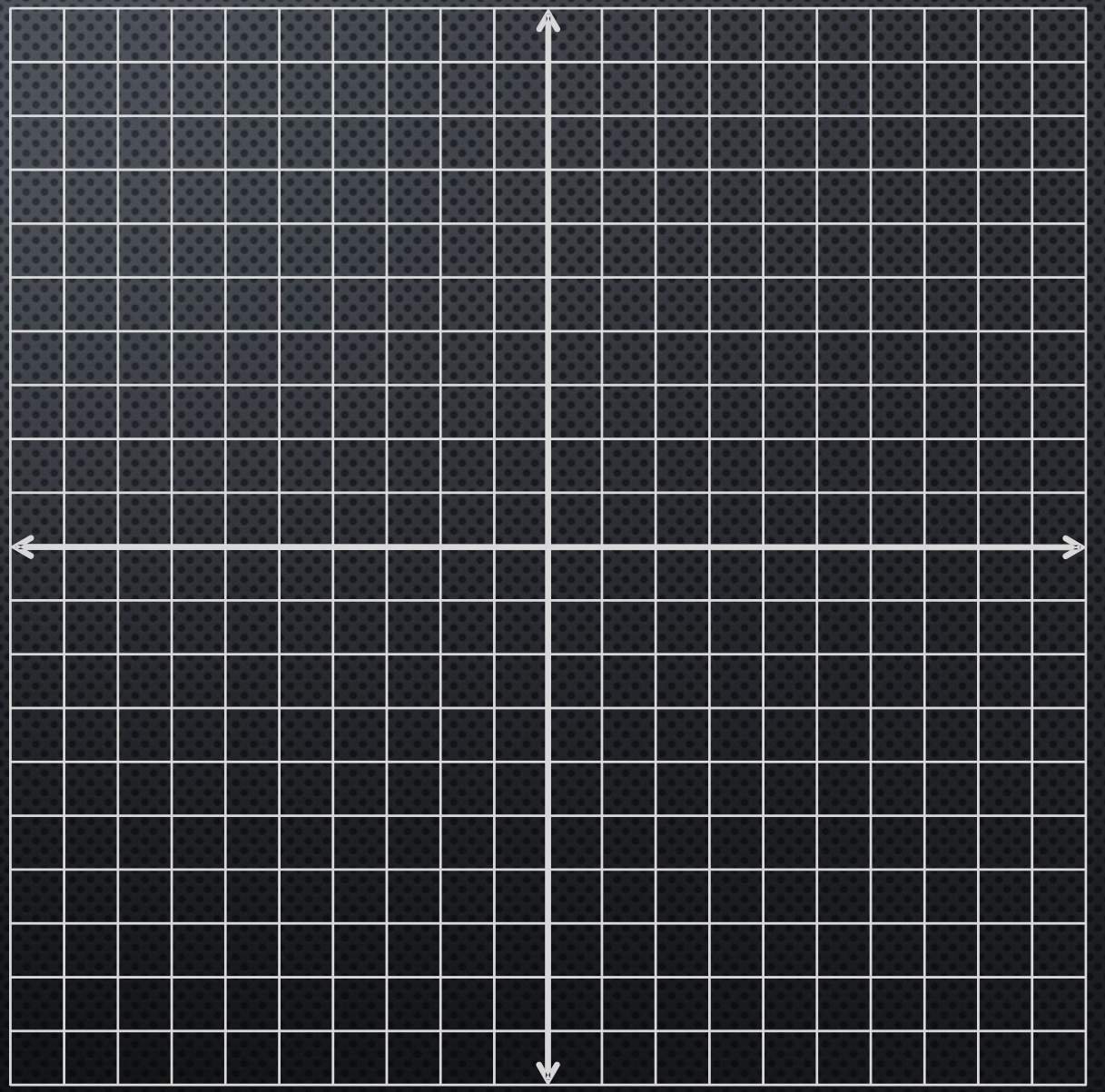
1-07 TRANSFORMATIONS OF FUNCTIONS

- Put it all together

  - a = vertical stretch
  - $\quad$ = horizontal stretch
  - c = horizontal shift right
  - d = vertical shift up

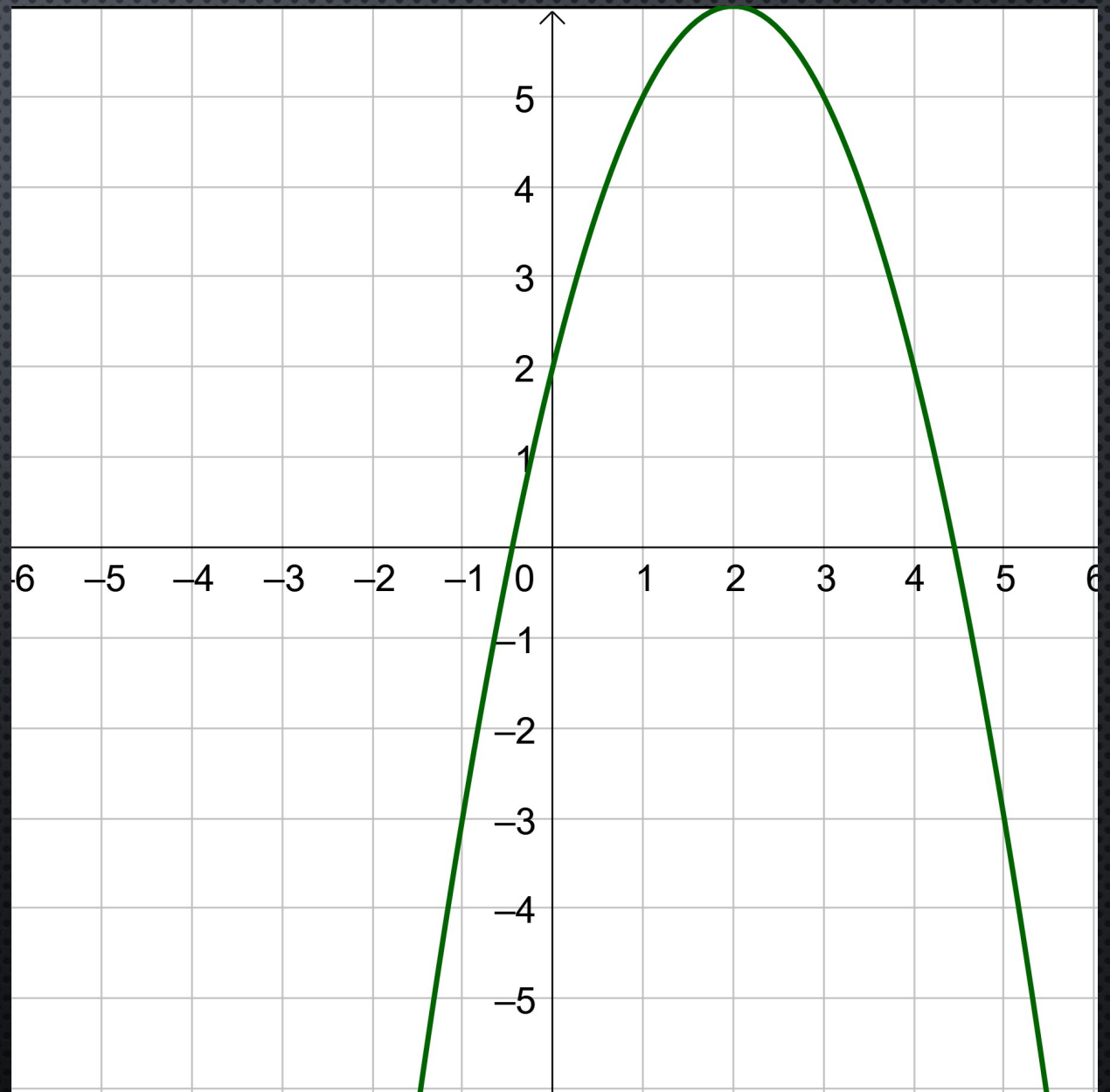
1-07 TRANSFORMATIONS OF FUNCTIONS

- Given
- Identify the parent function
- Describe the transformations
- Sketch the graph
- Use functional notation to write g in terms of f



1-07 TRANSFORMATIONS OF FUNCTIONS

- Write the function for



1-08 COMBINATIONS OF FUNCTIONS

In this section, you will:

- Combine functions using algebraic operations.
- Create a composition of functions.

1-08 COMBINATIONS OF FUNCTIONS

- Add
- Subtract
- Multiply
- Divide
  -

- If and , find

1-08 COMBINATIONS OF FUNCTIONS

- Composition
- Substitute g into f
- If and , find
- $f \circ g$
- $g \circ f$
- Domain of is all x in domain of g such that is in the domain of f.
- $x \rightarrow g \rightarrow f$
- If and , find the domain of $f \circ g$

1-08 COMBINATIONS OF FUNCTIONS

- Decompose
- Find and so that
- Pick a portion to be g(x), then replace that with x to get f(x)
- Decompose
- Decompose

1-09 INVERSE FUNCTIONS

In this section, you will:

- Verify that two functions are inverse functions.
- Find the domain and range on inverse functions.
- Find the inverse of a function.

1-09 INVERSE FUNCTIONS

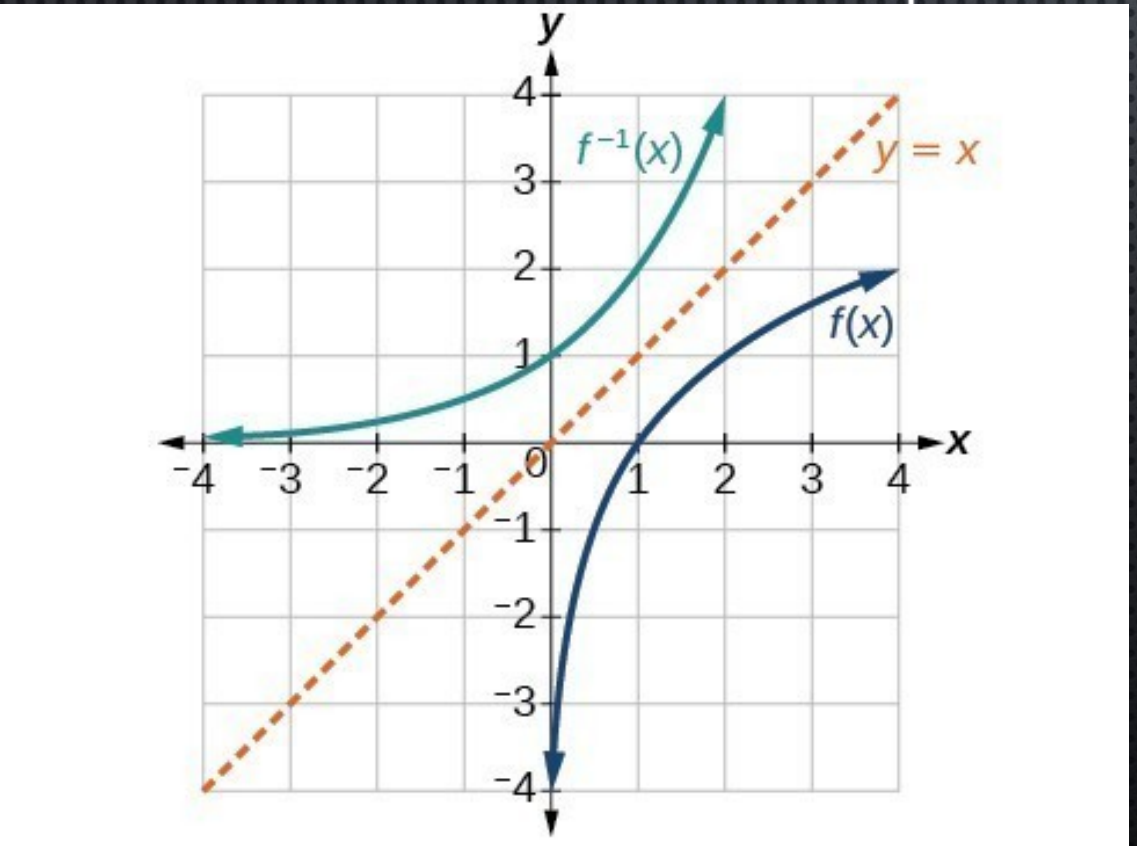
- Inverse functions
- Switch x and y
- Switch inputs and outputs

- Verify inverses by showing
- and

- Verify that and are inverses

1-09 INVERSE FUNCTIONS

- Graphs of inverses
- Reflected over line $y = x$



- One-to-one
- A function is one-to-one if each y corresponds to exactly one x.
- Passes the horizontal line test
- Inverse of a 1-to-1 is a function

1-09 INVERSE FUNCTIONS

- Finding inverses

1. Replace $f(x)$ with y
2. Switch x and y
3. Solve for y
4. If you did step 1, replace y with

- Find the inverse of

1-09 INVERSE FUNCTIONS

- Find the inverse of

1-10 MATHEMATICAL MODELING

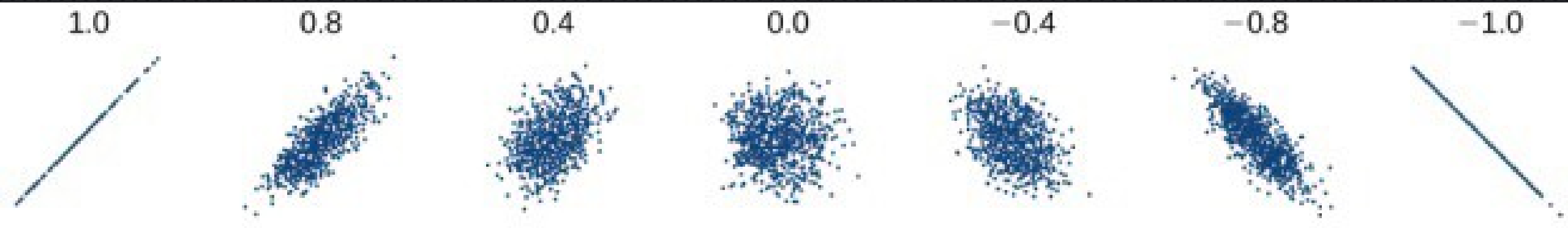
In this section, you will:

- Draw and interpret scatter plots.
- Find the best-fitting line using a graphing utility.
- Calculate variations.

1-10 MATHEMATICAL MODELING

- Mathematical modeling
- Find a function to fit data points

- Least squares regression (linear)
- Gives the best fitting line
- The amount of error is given by the correlation coefficient (*r*)



1-10 MATHEMATICAL MODELING

- Number (in 1000s) of female USAF personnel, P, on active duty

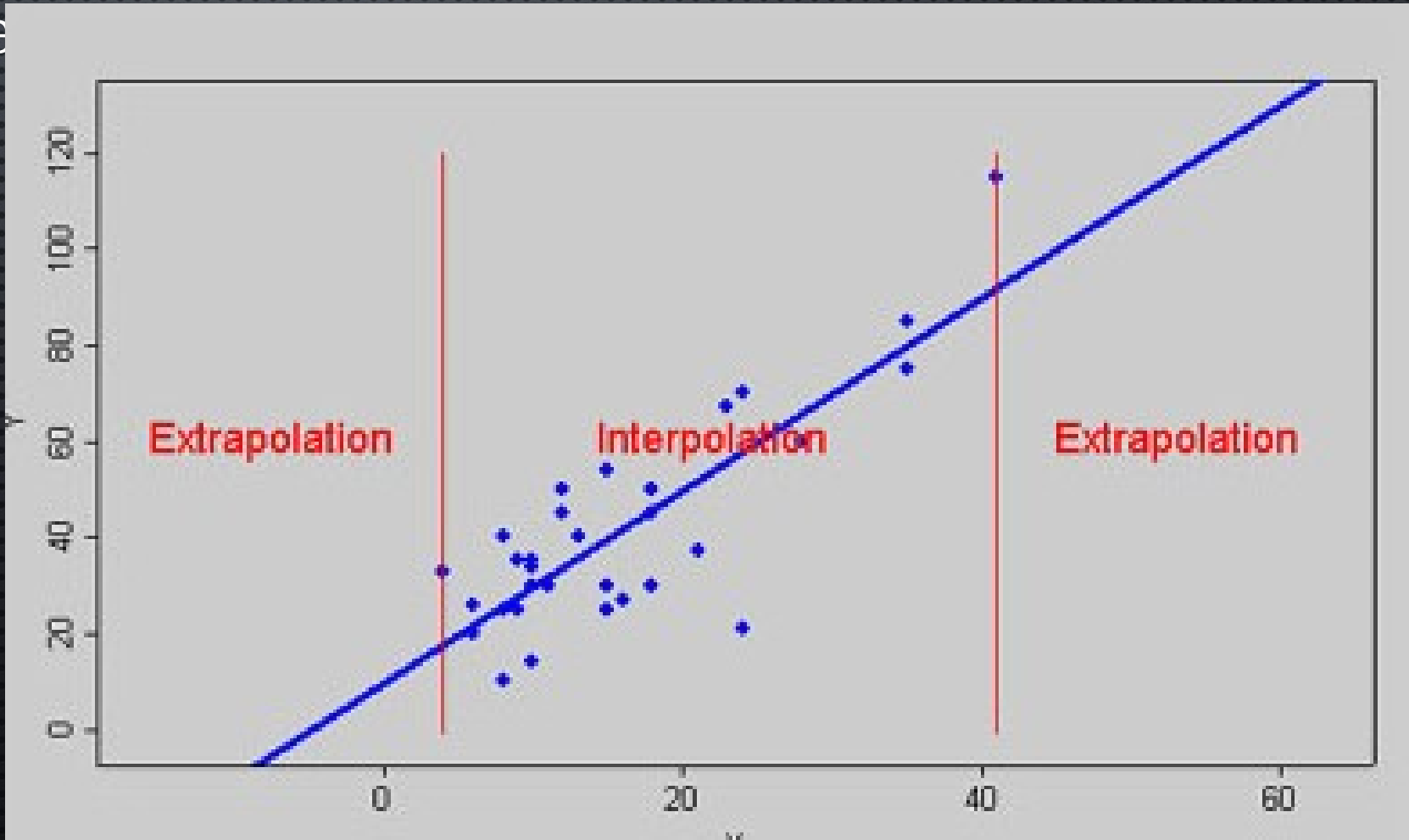
Year	2000	2001	2002	2003	2004
P	66.8	67.6	71.5	73.5	73.8

- Find a model with t=0 being 2000
- On TI-graphing
- STAT ∨ Edit… and enter data
- STAT → CALC ∨ LinReg(ax+b)

1-10 MATHEMATICAL MODELING

- Real-Life Problems
- Slope = rate of change

- Interpolation
  - Within data
  - Small error
- Extrapolation
  - Outside of data
  - Possibly huge error



1-10 MATHEMATICAL MODELING

- Variations
- Direct
- Inverse
- Joint
- a = constant of variation

- A company found the demand for its product varies inversely as the price of the product. When the price is $2.75, the demand is 600 units. Write an equation.